Time-series,
Spring, 2026

Autocorrelation Function (ACF) & Correlogram

Faculty of DS & AI
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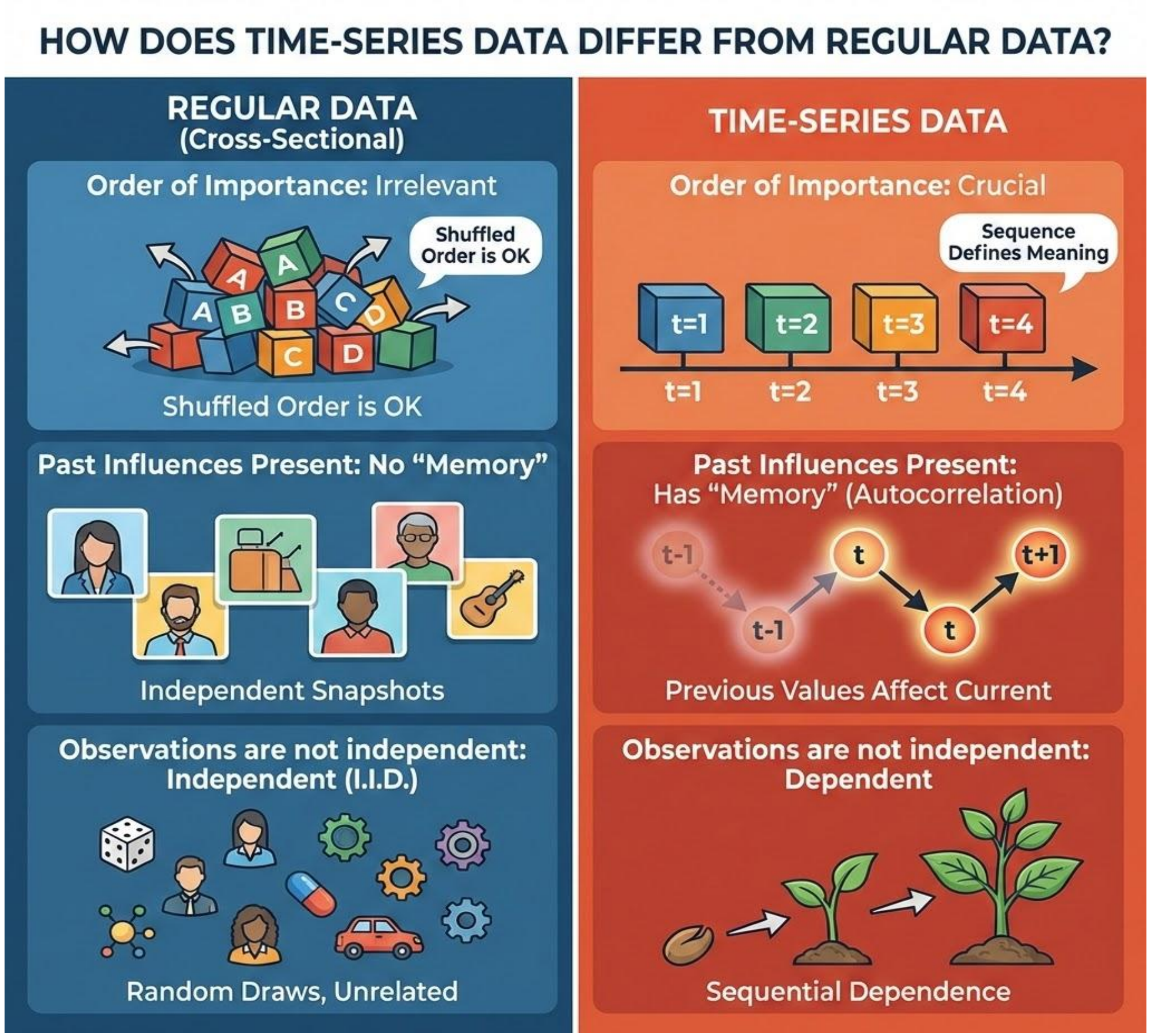
Business AI Lab

Content

- The core issue of the Time-series

The core issue of the Time-series

In time-series data:
- Order of Importance
- Past influences present
- Observations are not independent

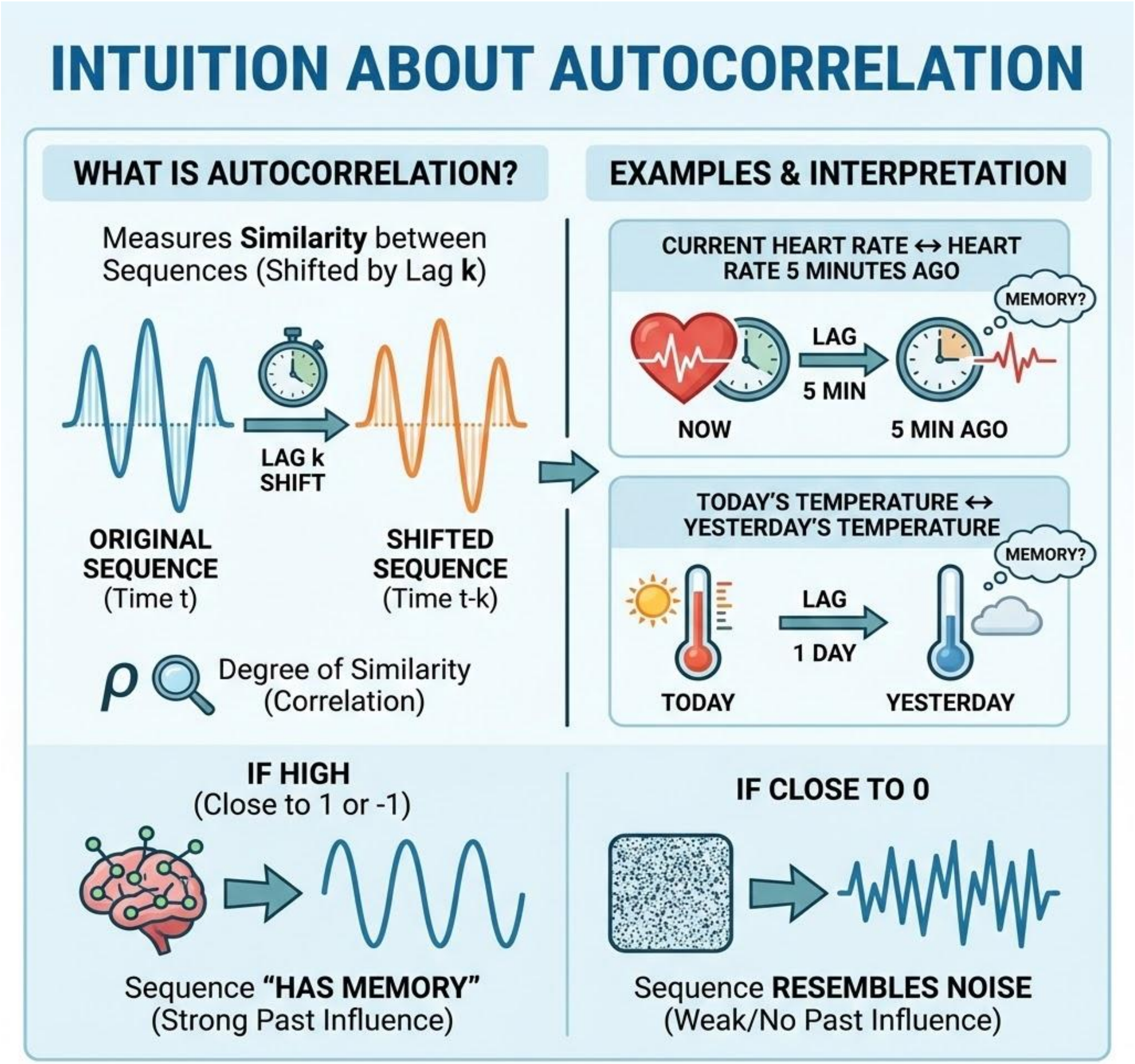


The core issue of the Time-series

Intuition about Autocorrelation

- Autocorrelation measures:
  - The **degree of similarity** between sequences
  - When a **sequence is shifted by k time steps (lag k)**
- Examples:
  - Current heart rate ↔ heart rate 5 minutes ago (lag 5 minutes)
  - Today's temperature ↔ yesterday's temperature (lag 1 day)

  If:
  - High → the sequence "has memory"
  - Close to 0 → the sequence resembles noise



The core issue of the Time-series

Autocorrelation

Let's assume we have a time series represented by the formula:

$$\{x_t\}_{t=1}^{T}$$

Mean:

$$\mu = \mathbb{E}[x_t]$$

Variance:

$$\sigma^2 = \mathbb{E}[(x_t - \mu)^2]$$

With Lag values: $k = 0, 1, 2, \ldots$

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With Lag values: $k = 0, 1, 2, ...$

Autocovariance at lag k:

$$\gamma(k) = \mathbb{E}[(x_t - \mu)(x_{t-k} - \mu)] \quad (1)$$

Autocorrelation Function (ACF)

$$\rho(k) = \frac{\gamma(k)}{\gamma(0)} \quad (2)$$

Where:

- $\gamma(0) = \sigma^2$
- $-1 \leq \rho(k) \leq 1$

The core issue of the Time-series

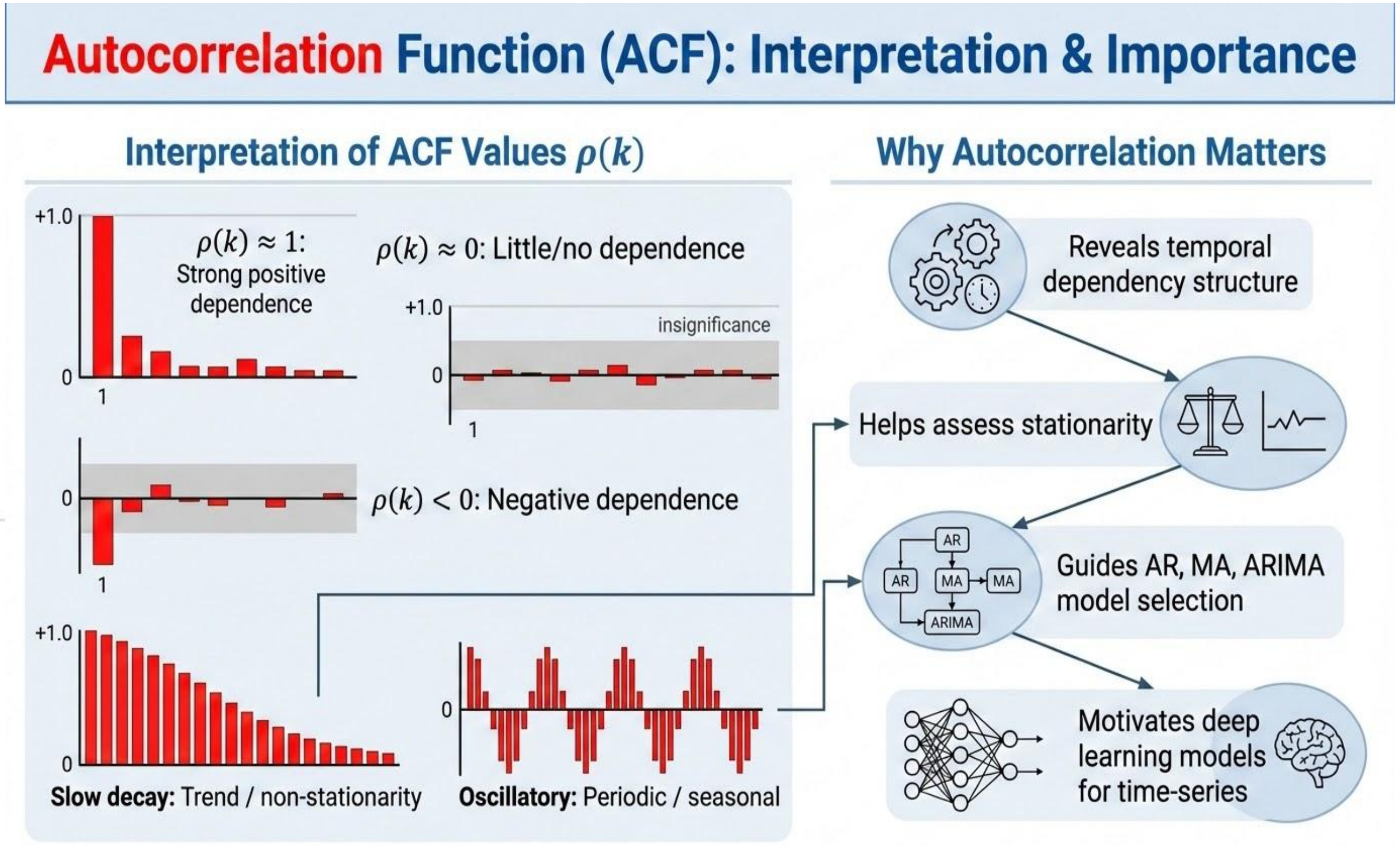
Autocorrelation

Interpretation of ACF Values $\rho(k)$

- $\rho(k) \approx 1$: Strong positive temporal dependence
- $\rho(k) \approx 0$: Little or no temporal dependence
- $\rho(k) < 0$: Negative temporal dependence
- Slow decay of ACF: Trend / non-stationarity
- Oscillatory ACF: Periodic or seasonal patterns

Why Autocorrelation Matters

- Reveals temporal dependency structure
- Helps assess stationarity
- Guides AR, MA, ARIMA model selection
- Motivates deep learning models for time-series

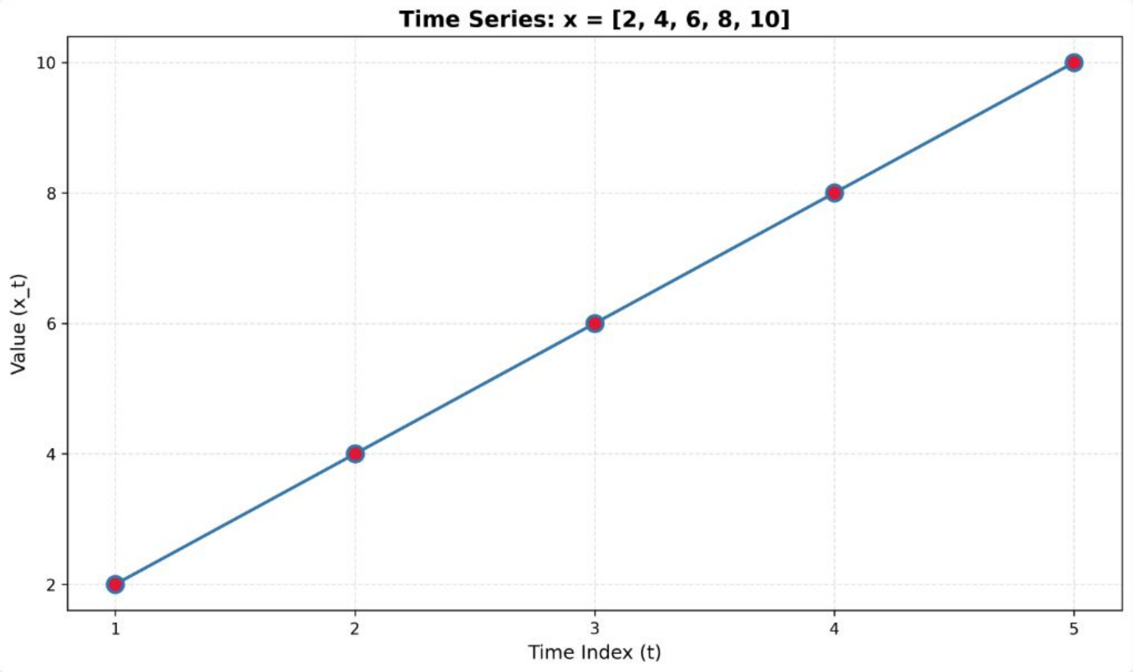


The core issue of the Time-series

Exercises

Problem 1.1

Given the time series: $x = [2, 4, 6, 8, 10]$



Compute:

1. The sample mean $\bar{x}$
2. The sample variance s^2
3. The autocovariance $\gamma(k)$ for lags $k = 0, 1, 2$
4. The autocorrelation $\rho(k)$ for lags $k = 0, 1, 2$

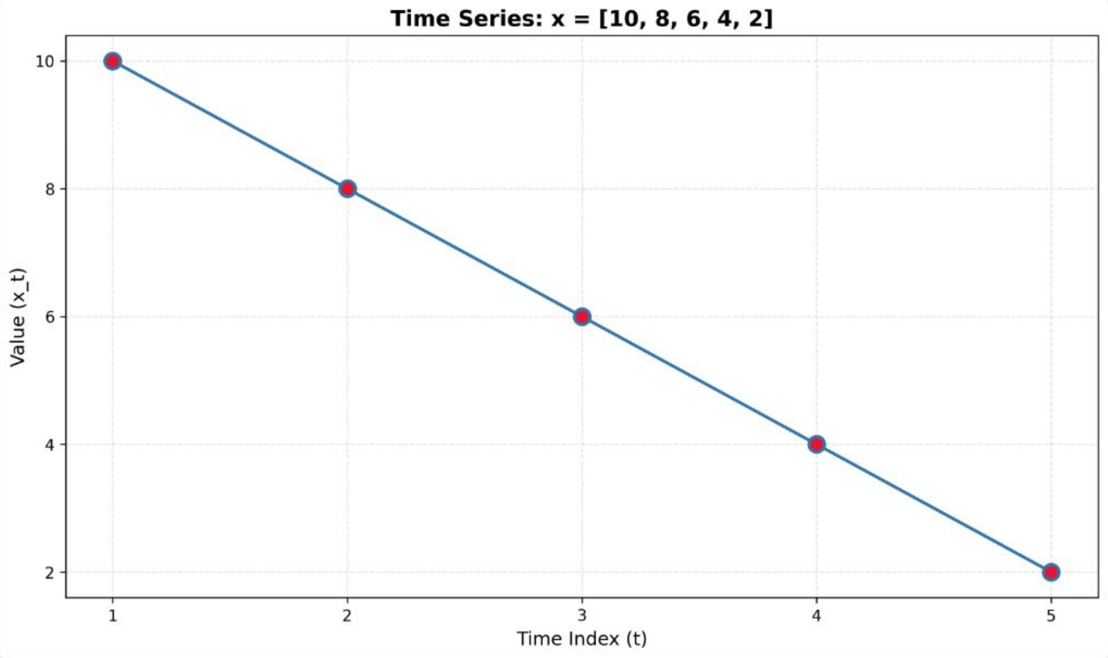
Problem 1.2

Given the time series: $x = [5, 5, 5, 5, 5]$

Compute the autocovariance and autocorrelation. What happens when all values are identical?

Problem 1.3

Given the time series: $x = [10, 8, 6, 4, 2]$



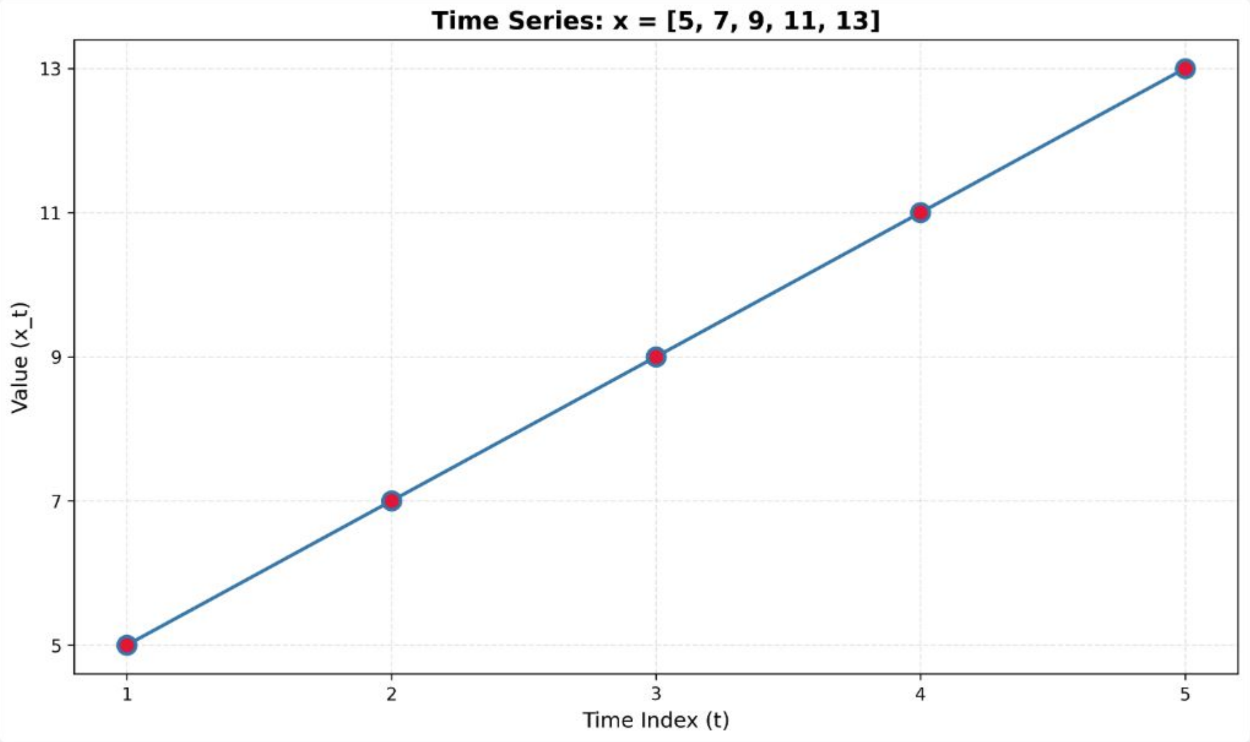
Compute the autocovariance $\gamma(k)$ and autocorrelation $\rho(k)$ for lags $k = 0, 1, 2$.

The core issue of the Time-series

Exercises

Problem 2.1

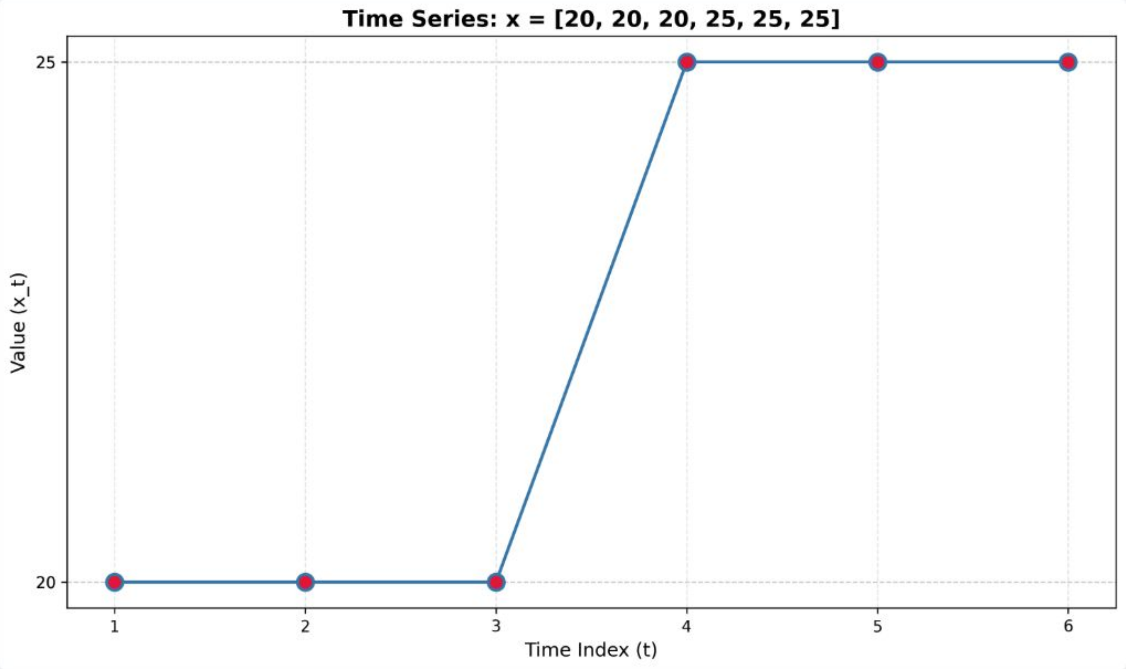
Given the time series: $x = [5, 7, 9, 11, 13]$



1. Compute the autocovariance **without** removing the mean.
2. Compute the autocovariance **with** mean removal.
3. Compare the results and explain why mean removal is necessary.

Problem 2.3

Given the time series: $x = [20, 20, 20, 25, 25, 25]$



Compare the autocorrelation at lag 1 computed with and without mean removal. What does this tell you about the series?

Problem 2.4

Given the time series: $x = [1, 3, 5, 1, 3, 5]$

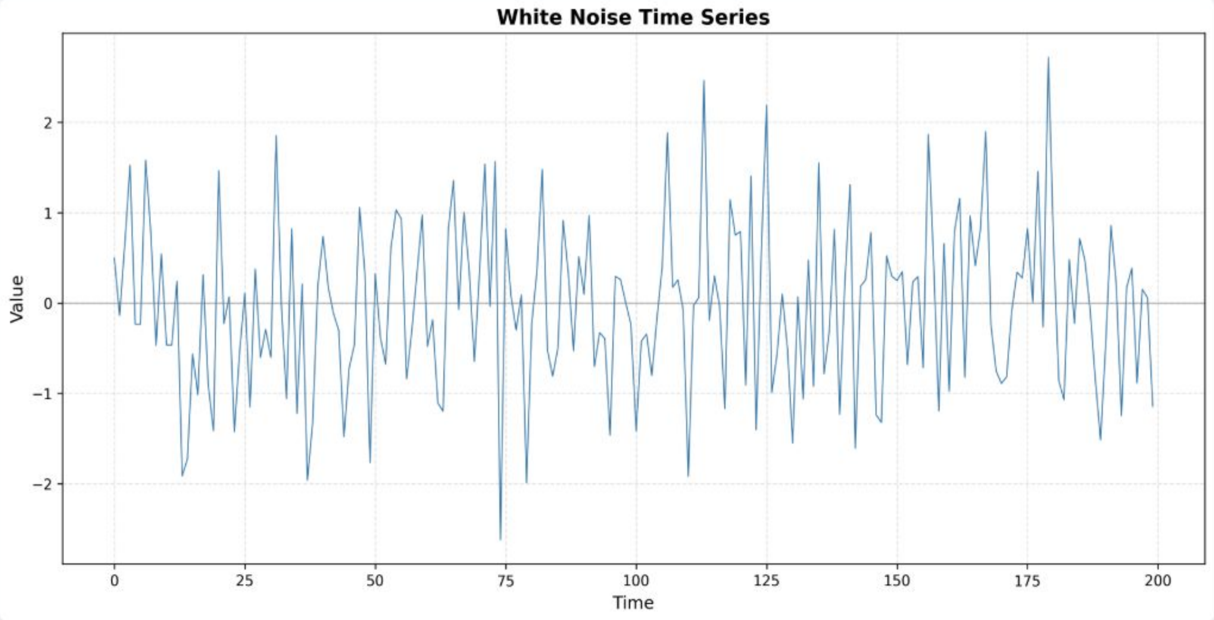
Compute autocorrelation at lag 2 with and without mean removal. Explain why the results differ.

The core issue of the Time-series

Exercises

Problem 3.1

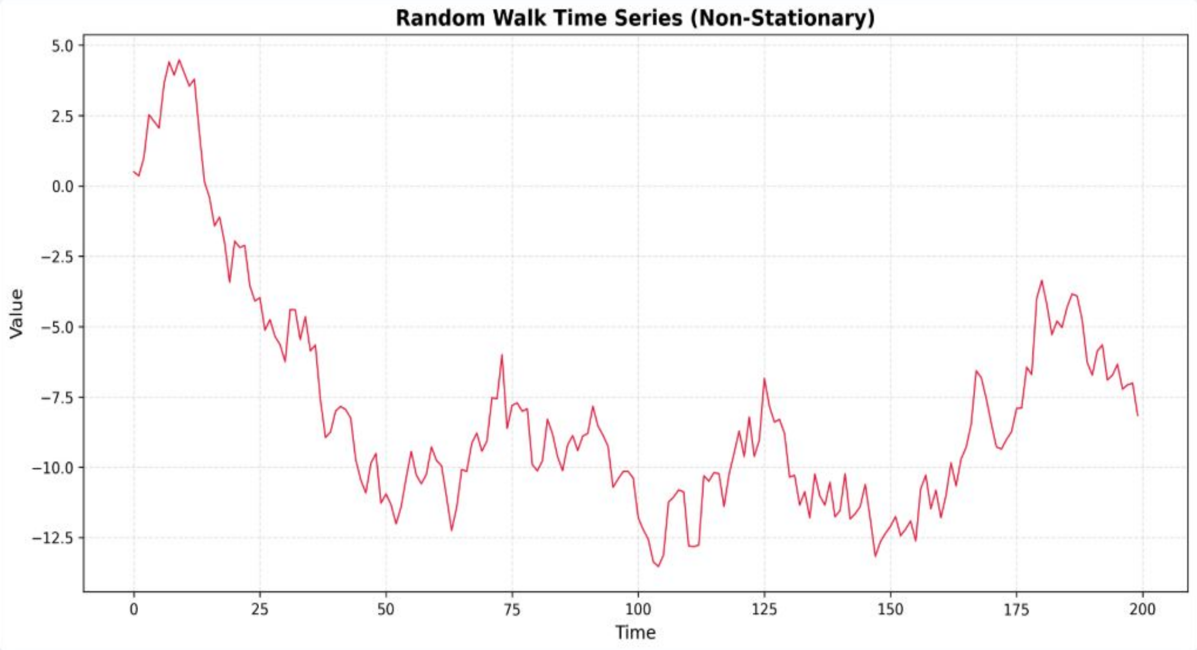
A time series has an ACF plot where $\rho(0) = 1$ and $\rho(k) \approx 0$ for all $k > 0$, with values randomly scattered around zero within the confidence bands.



1. What type of process does this indicate?
2. What are the characteristics of such a process?
3. Generate a synthetic time series with this ACF pattern and plot both the series and its ACF.

Problem 3.2

A time series has an ACF plot showing $\rho(k)$ that decays very slowly, remaining positive and significant even at large lags (e.g., $\rho(20) > 0.5$).



1. What does this pattern indicate?
2. What type of non-stationarity is likely present?
3. Generate a synthetic time series with this ACF pattern and plot both the series and its ACF.

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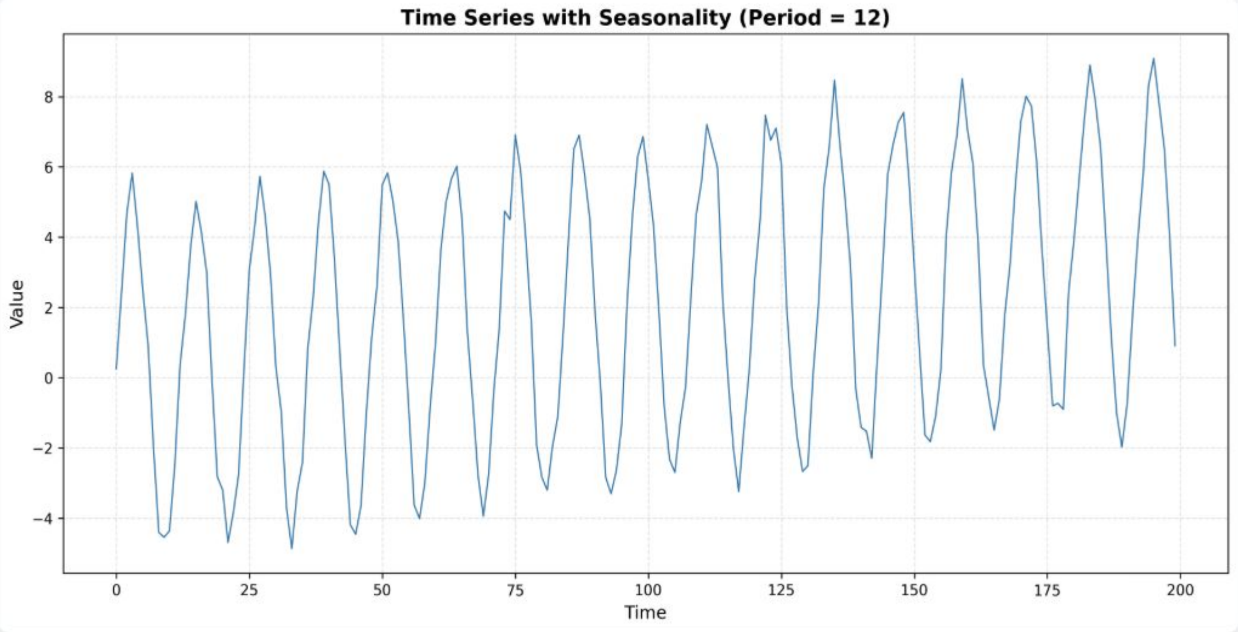
Problem 3.3

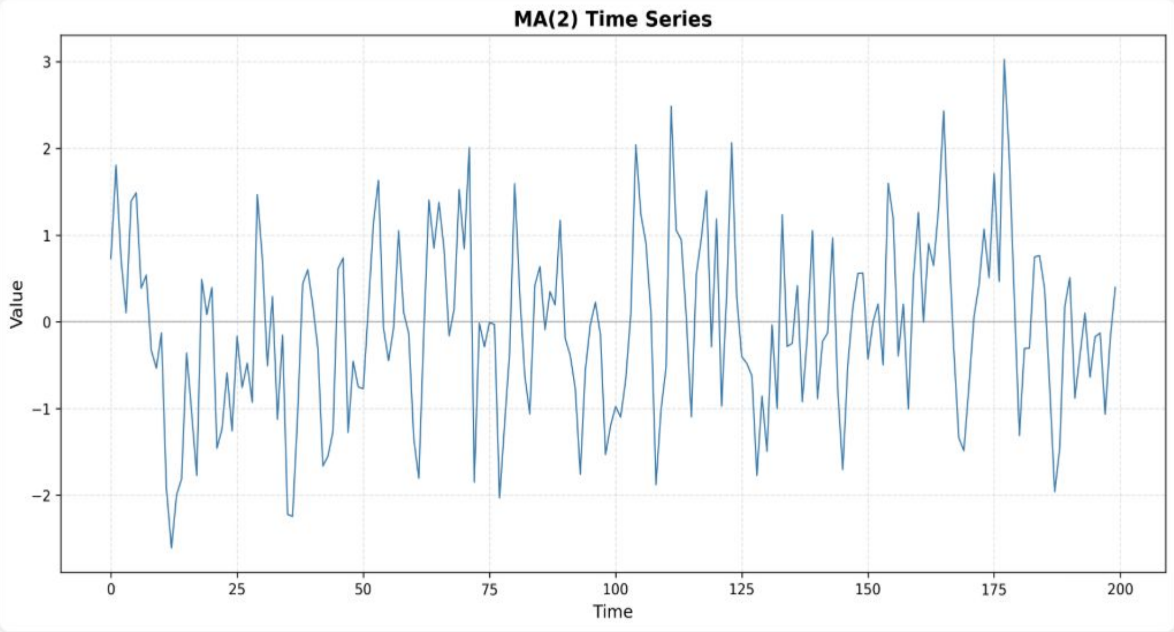
A time series has an ACF plot showing oscillatory (sinusoidal) behavior, with autocorrelations alternating between positive and negative values in a periodic pattern.



1. What does this pattern indicate?
2. What type of seasonality or cyclical behavior is present?
3. Generate a synthetic time series with this ACF pattern and plot both the series and its ACF.

Problem 3.4

A time series has an ACF plot where $\rho(k)$ shows a sharp cutoff after lag $q = 2$, with $\rho(1)$ and $\rho(2)$ being significant, but $\rho(k) \approx 0$ for all $k > 2$.



1. What type of process does this suggest?
2. What is the likely model order?
3. Generate a synthetic time series with this ACF pattern and plot both the series and its ACF.

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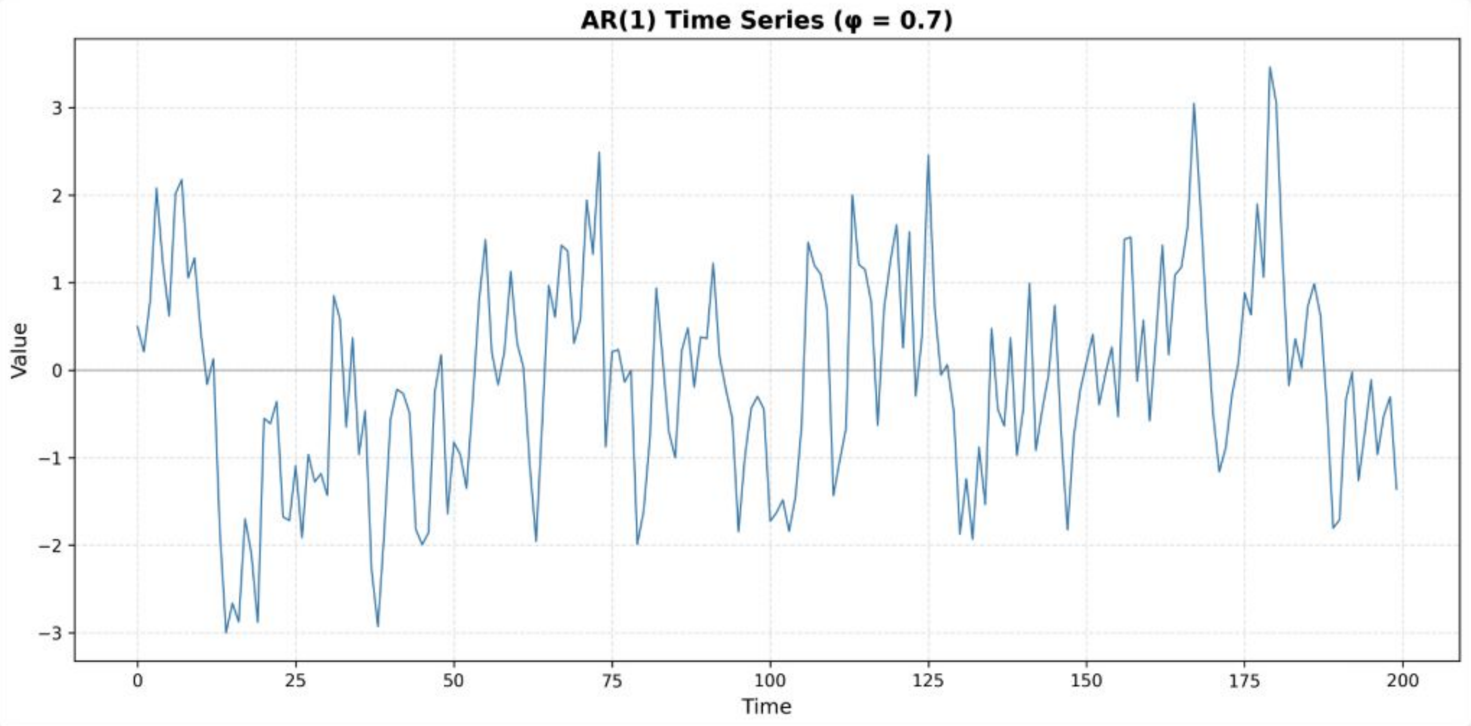
Exercises

Problem 3.5

A time series has an ACF plot showing exponential decay: $\rho(k)$ starts high and decays gradually, remaining positive but decreasing, with no sharp cutoff.



1. What type of process does this indicate?
2. How does this differ from the MA process pattern?
3. Generate a synthetic time series with this ACF pattern and plot both the series and its ACF.

Thank you!